The theoretical predictions for the study of the  $a_0(980)$  and  $f_0(980)$  mesons in the  $\phi$  radiative decays. \*

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## Abstract

The potentialities of the production of the  $a_0$  and  $f_0$  mesons in the  $\phi$  radiative decays are considered.

The central problem of light hadron spectroscopy has been the problem of the scalar  $f_0(980)$  and  $a_0(980)$  mesons. It is well known fact that these states possess peculiar properties from the naive quark  $(q\bar{q})$  model point of view, see, for example [1, 2, 3, 4]. To clarify the nature of these mesons a number of models has been suggested. It was shown that all their challenging properties could be understood [1, 2, 3, 4] in the framework of the four-quark  $(q^2\bar{q}^2)$  MIT-bag model [5] with symbolic quark structure  $f_0(980) = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$  and  $a_0(980) = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$ . Along with the  $q^2\bar{q}^2$  nature of  $a_0(980)$  and  $f_0(980)$  mesons the possibility of their being the  $K\bar{K}$  molecule is discussed [6]. During the last few years it was established [7, 8, 9] that the radiative decays of the  $\phi$  meson  $\phi \to \gamma f_0 \to \gamma \pi \pi$  and  $\phi \to \gamma a_0 \to \gamma \eta \pi$  could be a good guideline in distinguishing the  $f_0$  and  $g_0$  meson models. The branching ratios are considerably different in the cases of naive quark, four-quark or molecular models. As has been shown [7, 8, 9], in the four quark model the branching ratio is

$$BR(\phi \to \gamma f_0(q^2\bar{q}^2) \to \gamma \pi \pi) \simeq BR(\phi \to \gamma a_0(q^2\bar{q}^2) \to \gamma \pi \eta) \sim 10^{-4},$$
 (1)

and in the  $K\bar{K}$  molecule model it is

$$BR(\phi \to \gamma f_0(K\bar{K}) \to \gamma \pi \pi) \simeq BR(\phi \to \gamma a_0(K\bar{K}) \to \gamma \pi \eta) \sim 10^{-5}.$$
 (2)

<sup>\*</sup>Talk presented by V.V. Gubin

It is easy to note that in the case  $f_0 = s\bar{s}$  and  $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$  (so called  $s\bar{s}$  model [10]) the branching ratios  $BR(\phi \to \gamma f_0 \to \gamma \pi \pi)$  and  $BR(\phi \to \gamma a_0 \to \gamma \pi \eta)$  are different by factor of ten, which should be visible experimentally.

In the case when  $f_0 = s\bar{s}$  the suppression by the OZI rule is absent and the evaluation gives [7, 9]

$$BR(\phi \to \gamma f_0(s\bar{s}) \to \gamma \pi \pi) \simeq 5 \cdot 10^{-5},$$
 (3)

whereas for  $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$  the decay  $\phi \to \gamma a_0 \to \gamma \pi \eta$  is suppressed by the OZI rule and is dominated by the real  $K^+K^-$  intermediate state breaking the OZI rule [7, 9]

$$BR(\phi \to \gamma a_0(q\bar{q}) \to \gamma \pi \eta) \simeq (5 \div 8) \cdot 10^{-6}.$$
 (4)

Imposing the appropriate photon energy cuts  $\omega < 100$  MeV, one can show [9] that the background reactions  $e^+e^- \to \rho(\omega) \to \pi^0\omega(\rho) \to \gamma\pi^0\pi^0$ ,  $e^+e^- \to \rho(\omega) \to \pi^0\omega(\rho) \to \gamma\pi^0\eta$  and  $e^+e^- \to \phi \to \pi^0\rho \to \gamma\pi^0\pi^0(\eta)$  are negligible in comparison with the scalar meson contribution  $e^+e^- \to \phi \to \gamma f_0(a_0) \to \gamma\pi^0\pi^0(\eta)$  for  $BR(\phi \to \gamma f_0(a_0) \to \gamma\pi^0\pi^0(\eta))$  greater than  $5 \cdot 10^{-6}(10^{-5})$ .

Let us consider the reaction  $e^+e^- \to \phi \to \gamma(f_0 + \sigma) \to \gamma\pi^0\pi^0$  with regard to the mixing of the  $f_0$  and  $\sigma$  mesons. We consider the one-loop mechanism of the R meson production, where  $R = f_0, \sigma$ , through the charged kaon loop,  $\phi \to K^+K^- \to \gamma R$ , see [7, 8, 9]. The whole formalism in the frame of which we study this problem is discussed in [9]. The parameters of the  $f_0$  and  $\sigma$  mesons we obtain from fitting the  $\pi\pi$  scattering data, see [9].

In the four-quark model and  $s\bar{s}$  model we consider the following parameters to be free: the coupling constant of the  $f_0$  meson to the  $K\bar{K}$  channel  $g_{f_0K^+K^-}$ , the coupling constant of the  $\sigma$  meson to the  $\pi\pi$  channel  $g_{\sigma\pi\pi}$ , the constant of the  $f_0-\sigma$  transition  $C_{f_0\sigma}$ , the ratio  $R=g_{f_0K^+K^-}^2/g_{f_0\pi^+\pi^-}^2$ , the phase  $\theta$  of the elastic background and the  $\sigma$  meson mass. The mass of the  $f_0$  meson is restricted to the region 0.97  $< m_{f_0} < 0.99$  GeV. Treating the  $\sigma$  meson as an ordinary two-quark state, we get  $g_{\sigma K^+K^-}=\sqrt{\lambda}g_{\sigma\pi^+\pi^-}/2\simeq 0.35g_{\sigma\pi^+\pi^-}$ , where  $\lambda\simeq 1/2$  takes into account suppression of the strange quark production. So the constant  $g_{\sigma K^+K^-}$  (and  $g_{\sigma\eta\eta}$ ) is not essential in our fit.

As for the reaction  $e^+e^- \to \gamma \pi^0 \eta$  the similar analysis of the  $\pi \eta$  scattering cannot be performed directly. But, our analysis of the final state interaction for the  $f_0$  meson production show that the situation does not changed radically, in any case in the region  $\omega < 100$  MeV. Hence, one can hope that the final state interaction in the  $e^+e^- \to \gamma a_0 \to \gamma \pi \eta$  reaction will not strongly affect the predictions in the region  $\omega < 100$  MeV. Based on the analysis of  $\pi \pi$  scattering and using the relations between coupling constants we predict the quantities of the  $BR(\phi \to \gamma a_0 \to \gamma \pi \eta)$  in the  $q^2\bar{q}^2$  model,  $K\bar{K}$  model and the  $q\bar{q}$  model where  $f_0 = s\bar{s}$  and  $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ .

The fitting shows that in the four quark model  $(g_{f_0K^+K^-}^2/4\pi \sim 1~GeV^2)$  a number of parameters describe well enough the  $\pi\pi$  scattering in the region 0.7 <  $m < 1.8~{\rm GeV}$ , see [9]. We predict  $BR(\phi \to \gamma(f_0 + \sigma) \to \gamma\pi\pi) \sim 10^{-4}$  and  $BR(\phi \to \gamma a_0 \to \gamma\pi\eta) \sim 10^{-4}$  in the  $q^2\bar{q}^2$  model.

In the model of the  $K\bar{K}$  molecule we get  $BR(\phi \to \gamma(f_0 + \sigma) \to \gamma\pi\pi) \sim 10^{-5}$  and  $BR(\phi \to \gamma a_0 \to \gamma\pi\eta) \sim 10^{-5}$ .

In the  $q\bar{q}$  model the  $f_0(a_0)$  meson is considered as a point-like object, i.e. in the  $K\bar{K}$  loop,  $\phi \to K^+K^- \to \gamma f_0(a_0)$  and in the transitions caused by the  $f_0 - \sigma$  mixing we consider both the real and the virtual intermediate states. This model is different from  $q^2\bar{q}^2$  model by the coupling constant which is  $g_{f_0K^+K^-}^2/4\pi < 0.5~GeV^2$ . In this model we obtain  $BR(\phi \to \gamma(f_0 + \sigma) \to \gamma\pi\pi) \simeq 5 \cdot 10^{-5}$  and taking into account the imaginary part of the decay amplitude only, which violates the OZI rule, we get  $BR(\phi \to \gamma a_0(q\bar{q}) \to \gamma\pi\eta) \simeq 8 \cdot 10^{-6}$ .

The experimental data from SND and CMD-2 detectors support the four quark nature of the  $f_0$  and  $a_0$  mesons, see Fig.1 and Fig.2. and also [11, 12, 13, 14]. The obtained parameters for  $f_0$  meson from SND detector are  $m_{f_0} = 971 \pm 6 \pm 5$  MeV,  $g_{f_0K^+K^-}^2/4\pi = 2.1 \pm_{0.56}^{0.88} \ GeV^2$ , R = 4.1 and  $BR(\phi \to \pi^0\pi^0\gamma) = (1.14 \pm 0.1 \pm 0.12) \cdot 10^{-4}$ , see the dashed line on Fig.1.

As for reaction  $e^+e^- \to \gamma \pi^+\pi^-$ , the analysis shows that the study of this reaction is an interesting and rather complex problem.

The main problem is the large background process of final pions radiation. The  $f_0$  state in this reaction could be studied only by observing the interference patterns in the total cross-section and in the photon spectrum [15, 16]. As it was shown in [16], since the Fermi-Watson theorem for the final state interaction due to the soft photons in the reaction  $e^+e^- \to \rho(s) \to \gamma\pi^+\pi^-$  is not valid, the phase of the amplitude  $\gamma^*(s) \to \rho \to \gamma\pi\pi$  does not determined by the s-wave phase of  $\pi\pi$  scattering. The analyses of the interference patterns in the reaction  $e^+e^- \to \phi + \rho \to \gamma f_0 + \gamma \pi^+\pi^- \to \gamma \pi^+\pi^-$  should be performed taking into account the phase of the elastic background of the  $\pi\pi$  scattering, the phase of the triangle diagram  $\phi \to K^+K^- \to \gamma f_0$  and the phase of the  $f_0$ - $\sigma$  complex in the  $\phi \to K^+K^- \to \gamma(f_0 + \sigma) \to \gamma\pi\pi$  amplitude. The whole formalism for the description of these reactions and the resulting pictures were stated in [9, 15, 16].

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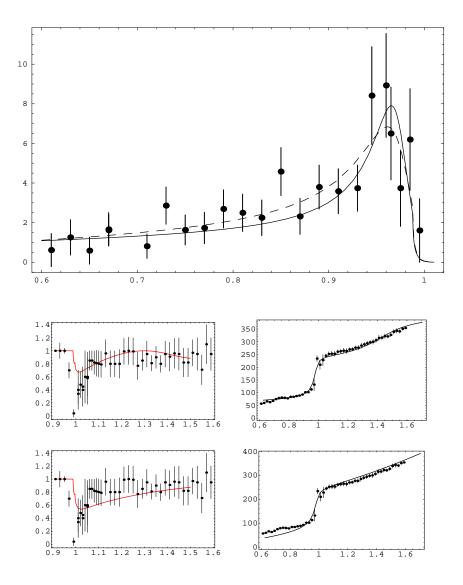


Figure 1: The simultaneous fit of the spectrum of the differential cross section  $d\sigma(e^+e^- \to \gamma(f_0 + \sigma) \to \gamma\pi^0\pi^0)/d\omega$  with mixing of the  $f_0$  and  $\sigma$  mesons (solid line) and of the  $\pi\pi$  scattering data (first row). The branching ratio for this fit is  $BR(\phi \to \gamma\pi^0\pi^0) = 2.8 \cdot 10^{-4}$ . The dashed line is the spectrum of the  $f_0$  meson without mixing with the  $\sigma$  meson. The  $\pi\pi$  scattering for this fit is in the second row.

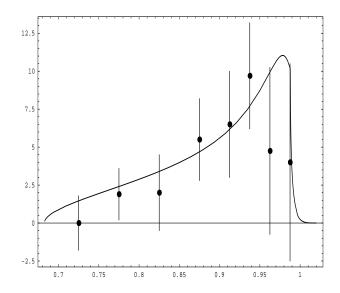


Figure 2: The fit of the spectrum of the differential cross section  $d\sigma(e^+e^- \to \gamma a_0 \to \gamma \pi \eta)/d\omega$ . Parameters of the fit are  $m_{a_0} = 986\pm^{23}_{10}$  MeV,  $g^2_{a_0K^+K^-}/4\pi = 1.5\pm 0.5~GeV^2$  and  $BR(\phi \to \gamma \pi \eta) = (0.83\pm 0.23)\cdot 10^{-4}$ .

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